# MODELLING AND ANALYSIS OF MAGNETOHYDRODYNAMIC NON-NEWTONIAN FLUID FLOW THROUGH POROUS MEDIA

**A SYNOPSIS**

*Submitted by*

**GEETHA R**
**(Reg. No. 9210025001)**

*in partial fulfillment for the award of the degree of*

DOCTOR OF PHILOSOPHY

**DEPARTMENT OF MATHEMATICS**

**SCHOOL OF ADVANCED SCIENCES**

**KALASALINGAM ACADEMY OF RESEARCH AND EDUCATION**

(DEEMED TO BE UNIVERSITY)

**ANAND NAGAR, KRISHNANKOIL – 626 126**

**TAMIL NADU, INDIA**

**MAY 2024**

# 1. INTRODUCTION

A fluid is a substance that continuously deforms when shear stress is applied. Liquids, gases, plasmas, and plastic solids are considered fluids. Fluids are substances with zero shear modulus. The study of fluid is very important in every branch of science like physics, chemistry, biology and mathematics and also in many branches of engineering like chemical engineering, metallurgy along with in medical science. The fluid under study may be flowing in pipe or in channel, in pump or in compressor, around an aircraft or a missile, in an ocean or in an atmosphere.

Fluid dynamics is a sub discipline of fluid mechanics that deals with fluid flow, the nature science of fluid (liquids and Gases) in motion. The advancement in the subject of fluid dynamics was started in 1755 when Euler gave his famous equations of fluid flow for ideal (inviscid) fluids in his paper entitled "General principles of the motion of fluids". The fluid dynamics has several sub disciplines itself, including (i) Aerodynamics (the study of air and the other gases in motion). (ii) Hydrodynamics (the study of liquids in motion). Fluid dynamics is governed by the law of conservation which states that the total amount of energy, mass and linear momentum in closed system remain constant and that energy and mass can neither be created nor destroyed. All of these phenomena are governed by the compressible Navier-Stokes equations and the solutions of fluid dynamic problems are usually associated with the behavior of fluid properties. Fluid dynamics has a wide range of applications including forces and moments on aircraft, determining the mass flow rate of petroleum through pipeline, predicting weather patterns, understanding nebulae in interstellar space and modelling fission weapon detonation. Some of its principles are even used in traffic engineering, where traffic is treated as continuous fluid and crowd dynamics.

## 1.1 Types of Fluids

Fluids can be classified into several types based on their properties and behaviour. Some common types include:

**1.1.1 Newtonian Fluid:** Newtonian fluid is the one in which local rate of strain at every point is directly proportional to viscous stresses rising from its flow. In addition, A Newtonian fluid is fluid that has consistent thickness at an unfaltering temperature. Ex: Water, oil, fuel, liquor and glycerine.

**1.1.2 Non-Newtonian Fluid:** The fluids which don't follow the principles of Newton's law of viscosity can be classified as non-Newtonian fluids. In non-Newtonian fluids, the local rate of strain at every point is not directly proportional to viscous stresses rising from its flow. In general, non-Newtonian fluids are unpredictable blends such as slurries, pastes, gels, polymer solutions. The non-Newtonian fluids are extensively utilized for various aspects in several applications including food preservation techniques, power engineering, chemical process industries, and petroleum production. To elucidate the characteristics of non-Newtonian fluid flows several analytical and numerical methods have been proposed in the previous couple of decades.

## 1.2 Boundary Layer

Prandtl has determined the flow field far away from the solid boundary by considering viscosity of the fluid. The Navier-Stokes equations are suitable to characterise the transport phenomena of fluid with small values of Reynolds number while the associated non-linear terms in turn reduce the accuracy in predicting the characteristics of the fluid with high Reynolds number values. It is to be noted that the fluid viscosity plays a vital role in the region where the fluid flow very near to the solid surface. The momentum boundary layer is a thin layer which is formed when the fluid flow past a solid body or the fluid flow over the solid body. In several real-world problems, the governing equations which represent the boundary layer characteristics are second order, non-linear, elliptic type partial differential equations with boundary conditions. The ratio between inertial forces and viscous forces is known as Reynolds number which is very significant on describing the characteristics of the boundary layer. The fluid flow is called as laminar flow when the Reynolds number values are small whereas the fluid flow is known as turbulent flow when the Reynolds number values are high. It is to be inferred that the boundary layer is dominated by the viscous forces in case of laminar flows while the boundary layer is dominated by the inertia forces in case of turbulent flows. The types of boundary layers include momentum boundary layer, thermal boundary layer and solute boundary layer.

## 1.3 Magnetohydrodynamics (MHD)

Magnetohydrodynamics or magneto-fluid dynamics or hydro magnetics is the combination of fluid mechanics and electromagnetism. It describes how an

electrically conducting fluid responds to magnetic and electric fields. MHD is the idea that magnetic fields can cause currents in conductive fluids that are in motion, which in turn exert forces on the fluid and alters the magnetic field. Faraday was the first to identify a magnetic field interaction between sea flows and the Earth (1832). Hannes Alfven founded the field of MHD, for which, in 1970, he was awarded the Nobel prize in physics. The set of equations that describe magnetohydrodynamics is the combination of Navier-Stokes equations for fluid dynamics and Maxwell's equations for magnetism. These differential equations had to be solved concurrently, either analytically or numerically. The magnetohydrodynamic (MHD) equations are used to investigate the interaction between the bloodstream's magnetic fluid particles and the external magnetic field.

### 1.4 Heat Transfer

Heat is the energy which transferred from one system to another system due to temperature differences. It produces a feeling of warmth. Heat always travels from a medium with a higher temperature to one with a lower temperature and stops the transfer of heat when the two mediums reach the same temperature. A thermodynamic study represents how much heat is transferred as a system transition from one state of equilibrium to another. Heat transfer takes place by conduction, convection, and radiation. These three modes of heat transfer need a temperature difference and all modes move heat energy from a higher temperature to a lower temperature.

### 1.5 Mass Transfer

The net transport of mass in liquid and gaseous mediums from one point to another is referred to as a mass transfer. The element moves in a direction that lowers the gradient of concentration. Transport occurs from a higher concentration area to a lower concentration area. The gradient will be zero when equilibrium is reached. A mass transfer is the movement of a single ingredient from a higher to a lower concentration region. Mass transfer can occur in two different ways: Diffusion mass transfer, and Convective mass transfer.

### 1.6 Nanofluid

Nanofluids are fluids that are created through a laboratory preparation process that involves suspending nano-sized (1-100 nm) particles in basic fluids like water,

kerosene, methylene glycol, etc. These nanofluids were introduced by (Choi and Eastman, 1995). Nanofluids have been used recently in a variety of engineering and technological applications, including microprocessors to increase heat transmission rates as much as possible through collisions between different types of nanoparticles and to keep equipment cooled. In recent decades, researchers, engineers, and scientists have paid close attention to nanoparticles because of their size and form. Equipment can be cooled down more quickly through this process if the shape of the nanoparticles is spherical because they roll out in the fluid and transmit heat from higher energy molecules to lower heat capacity molecules very quickly. This is because the heat transmission rate is significantly higher than that of regular fluids.

### 1.7 Hybrid Nanofluids

Hybrid nanoparticles are nanoparticles made up of two or more distinct nanometer-sized components. Fluids made using hybrid nanoparticles are referred to as hybrid nanofluids.

### 1.8 Porous Medium

The term “porous medium” is used to describe a solid material that is made up of microscopic holes, or pores, that are randomly dispersed throughout the material. And among these pores, which allow the fluid can pass through them are effective pores which supports to the material porosity. While some pores among which do not allow the fluid to pass through them are ineffective pores of the solid. A region of space engaged by heterogeneous should not be a solid, but it may be in liquid form or gaseous state known as void, remaining portion called solid matrix. The best illustrations are like leather, sponge, wiper and ground soil and so on.

### 1.9 Chemical Reaction

In a chemical reaction process, reactants undergo a chemical reaction and transform into products through chemical transformation. In this, the bonds within the reactant molecules are broken, and new bonds are created within the product molecules to create a new chemical.

### 1.10 Soret Effect

The term Soret (thermal-diffusion) effect refers to the mass flux produced by a temperature gradient. Charles Soret in 1879 established that a salt solution existing in a cylindrical container at the two ends with different temperatures will have distinct composition at both ends. Near the cold end, the salt concentration was higher than at the hot end of the cylindrical container. He concluded from this that a temperature gradient caused a flux of salt.

The ratio of thermal and ordinary diffusion coefficients is referred as the Soret number. Generally, the Soret effect is of a lower order of magnitude and is frequently ignored during the mass transfer process. However, the Soret effect influences mass transfer for steep temperature gradients.

### 1.11 Dufour Effect

The flux of energy generated by a concentration gradient as a combined consequence of irreversible processes is the diffusion-thermo (Dufour) effect. When two dissimilar chemically non-reacting liquids or gases were allowed to diffuse one into another while both are at the same temperature, then there is a temperature difference. If the concentration is under control, the temperature difference persists. The Dufour effect was discovered to be of such magnitude in favour of the medium molecular weight (air, $N_2$), making it impossible to ignore.

## 2. LITERATURE REVIEW

Non-Newtonian fluids may be classified as pseudoplastic Williamson fluids. As its practical uses and inherent interest grow, the field of research known as boundary layer flow in pseudoplastic fluids is growing in prominence. The power law model, the Carreaus model, the Cross model, and the Ellis model are only some of the better examples of mathematical frameworks that may be used to explain the behaviour of pseudoplastic fluids. Williamson studied the behaviour of pseudoplastic fluids and proposed an equation-based model to define their motion in 1929. There have been empirical confirmations of this idea ever since. Williamson fluid peristaltic flow in a vertical annulus was investigated for its impact on heat and mass transmission by Nadeem and Akbar [1]. Vajravelu et al. [2] reported the Peristaltic transport of a Williamson fluid in asymmetric channels with permeable walls. The Williamson fluid model and its many uses in studying fluid dynamics are presented.

The Williamson fluid injection into a rock fracture perturbation solution was described by Dapra and Scrapi [3]. Alam Khan and Khan [4] investigated many approaches to four flow issues using a Williamson fluid by employing the homotopy analysis method (HAM). Williamson fluid flow over a linear and exponentially stretched surface has been modelled in two dimensions by Nadeem et al. [5]. Hayat et al. explored the time-independent MHD flow of Williamson fluid across a porous plate and found a solution with a number of phases by Hayat et al.[6]. The flow of a non-Newtonian fluid through a pipe with variable permeability and an inclined plane was taken into consideration by the authors Reddappa Bandi et al. [7]. Sudheer Babu et al. [8] investigated the impact of chemical reactions on MHD heat and mass transfer in a Jeffrey fluid flow with viscous dissipation and joule heating. They used a porous stretched sheet.

Nanofluids are increasingly widely employed in a wide range of industries because they include nanoparticles smaller than 100 nm, which improve heat conductivity. These sectors include the following: aggregate production, heat exchangers, solar collectors, pharmaceutical operations, reactors, absorption technologies, refrigerator chiller systems, and coolant applications [9–16]. Furthermore, new disciplines including brain tumour therapy, heart surgery, cancer treatment, and safe cooling techniques have found use for nanofluids. The groundwork for the research of nanofluids was laid by the groundbreaking studies of Masuda et al. [17] and Choi [18], which focused on the dispersion of ultrafine particles constantly suspended in a base fluid. Boundary layer flow and nanoparticle dispersion in a microchannel in the presence of a magnetic field [19-23].

The ambiguity of the boundary layer structure at two-channel walls that are inclined at a fixed angle to the flow of an incompressible, viscous fluid via a divergent/convergent channel is one of the major problems in fluid dynamics. Earlier studies on these difficulties were conducted by Jeffrey [24] and Hamel [25]. The existence of fluid volume sources and sinks at the intersection of two channel walls causes flow in certain situations, which can result in either a divergent or a convergent flow. For the velocity field, Pohlhausen [26] discovered an analytical convergent flow solution, while Millsaps and Pohlhausen [27] looked at the temperature distribution. Distinguished researchers Harison [28], Tollmien [29], Noether [30], and Dean [31] examined a few of this flow's interesting properties. Bhattacharyya and Layek [32] conducted more research on the MHD boundary layer of a power-law non-Newtonian

fluid in a diverging permeable channel with suction/injection. The MHD flow of a water-based Nanofluid in a stretching/shrinking convergent/divergent channel with thermal radiation effects was studied by Dogonchi and Ganji [33].

Some of the applications for studying natural flows in porous media include sedimentation, slurries, petroleum reservoir rocks, and sand beds. This has prompted several inquiries on Darcy's law and associated concepts related to porous media. Saravan and Shravan [34] examined the Soret effect of double diffusive convection in a Darcy porous medium. Alam and Rahman [35] expressed mixed convection flow as a function of suction and highlighted the Dufour and Soret effects using a vertical porous flat plate. Postelnicu [36] used natural convection to determine how the presence of a magnetic field impacts how quickly heat and mass are transferred from vertical surfaces in a porous material. Similarly, Alam et al. [37] and Alam and Rahman [38] conducted a similar study on the Dufour and Soret effects. A study by Afsar Khan et al. [39] focused on the peristaltic motion of a Jeffrey fluid flowing in a porous material in an asymmetric conduit as its viscosity changed. In their study, Reddappa et al. [40] discussed the Poiseuille flow of a Jeffrey fluid through parallel plates where one wall had a porous liner. Reddappa et al. [41] examined Couette flow of the same fluid through an inclined tube with permeable walls.

## 3. OBJECTIVES

This thesis addresses the heat and mass transfer problems of MHD non-Newtonian fluid with porous medium through various geometries. These problems can be handled either analytically/numerically depending on their complexity and feasibility of solution and interesting results are presented graphically with adequate physical interpretations. In the era of rapidly changing technologies, fluid mechanics has an important role to play. The typical complex flows encountered by the authors often comprise of two or more phases in which the interaction between them plays a dominant role in controlling transport processes such as heat and mass exchange and reaction kinetics. A quantitative study through a proper theory is essential to understand the physics of the complex flow behavior and also to obtain invaluable scale-up information for the industrial applications. In this context, we now present the objective and scope of the thesis.

## 4. RESEARCH PROBLEM

The objective of present thesis is to build up mathematical models systematically and numerically investigate the impact of heat and mass transfer on non-Newtonian Magnetohydrodynamic fluid flows within a porous medium through various geometries. The analysis considers a multitude of influential factors, including magnetic fields, radiation, porosity, Suction/blowing, non-Newtonian, and chemical reactions. The studies of such MHD non Newtonian flows in fluid mechanics have received enormous attention and find several applications in broad spectrum of disciplines like science, scientific engineering and technological fields because these flows are more prevalent and have large number of practical applications in nature. The subject is multidisciplinary and completely encircles in the view of applied mathematics and fluid mechanics ranging from chemical engineering to geophysics and the flow through porous media puts itself as major role in emphasizing the production of our basic needs such as oil, water and food.

### 4.1 Effect of Second Order Chemical Reaction on MHD Forced Convection Nanoparticles Cu, Ag, And $Fe_3O_4$ of Jeffrey Nanofluid over a Moving Plate in a Porous Medium in the Presence of Heat Source

The objective of this section is to determine how a second order chemical reaction will affect the magnetohydrodynamic flow of a water-based Jeffrey nanofluid containing Copper ( $Cu$ ), Silver ( $Ag$ ), and Ferrous Ferric Oxide ( $Fe_3O_4$ ) nanoparticles over a flowing stream embedded in a porous medium in the presence of a heat source/sink. The governing system of PDEs is converted into nonlinear ODEs using the similarity transformation approach, and these nonlinear ODEs are then resolved using MATLAB's built-in solver, bvp4c. In a single plot of three nanopartciles and tables, the output of the nanofluid velocity, temperature, and concentration are displayed for the included material properties as well as the related engineering physical parameters like coefficient of skin friction and rate of heat and mass transfer. This model has been rather successfully validated. The results indicate that the nanofluid $\theta(\eta)$ and $\Phi(\eta)$ profiles decline as $\lambda_1$ increases, while this behavior is opposite for the distributions of $f'(\eta)$. The kinetic energy of fluid particles decelerating because of the high viscosity. The suggested study demonstrates the significance of three nanoparticles in the disciplines of biocompatibility engineering

and medicines. Additionally, these nanoparticles are utilized in engineering, physics, space technology, operations involving high temperatures and cooling, pharmaceuticals, biosensors, paints, cosmetics, conductive coatings, and medical devices.

Figures 4.1(a), 4.1(b), 4.1(c), exhibits the exact effects of $(\lambda_1)$ for $Cu$, $Ag$ and $Fe_3O_4$ in water-based nanofluid on velocity $f'(\eta)$, temperature $\theta(\eta)$, and concentration profiles $\Phi(\eta)$. It is clear that $\theta(\eta)$ and $\Phi(\eta)$ declines as $(\lambda_1)$ increases, while this behaviour is opposite for the distributions of $f'(\eta)$. The kinetic energy of fluid particles is decelerating because of their high viscosity.

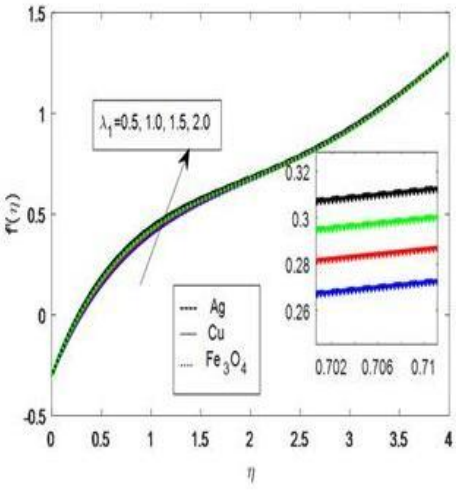


Figure 4.1 (a). Variation of $\lambda_1$ on velocity.

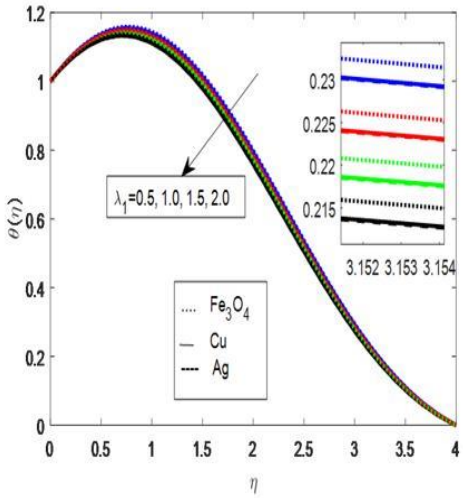


Figure 4.1 (b). Variation of $\lambda_1$ on temperature.

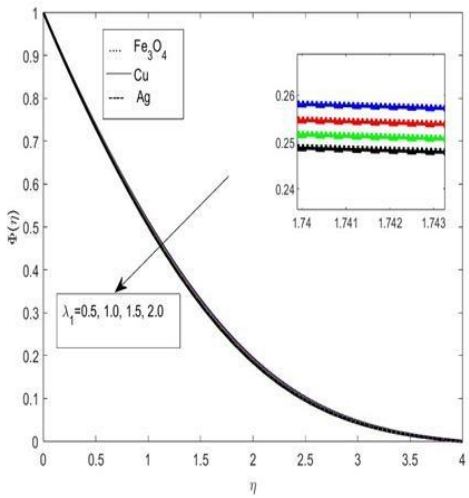


Figure 4.1 (c). Variation of $\lambda_1$ on concentration.

**4.2 Nonlinear Coupled PDEs for Nanofluid Flow and Heat Transfer with Varying Thickness: A Comparative Study**

The influences of heat production, thermal radiation, Brownian motion, thermophoresis, and chemical reactions were taken into consideration in the Jeffrey and Williamson model of nanofluid flow and heat transmission past a moving surface with changing thickness. Therefore, the purpose of this section is to investigate how a chemical reaction affects the MHD flow of a Jeffrey-Williamson nanofluid via a porous media that has adaptive thermal conductivity. Controlling PDEs can be converted to nonlinear coupled PDEs by using proper similarity analysis. These PDEs are solved with the required boundary conditions via the bvp4c technique. Graphs have been used by researchers to show how restrictions on dimensions other than space impact fluid velocity, temperature, and concentration. The properties of both the Jeffrey and Williamson nanofluids are discussed and compared. The Nusselt number, Sherwood number, and skin friction coefficient are calculated numerically. Comparing the current study's outcomes to those that have already been published for a particular case allows us to determine that there is great agreement between the two sets of data.

Figures 4.2(a), 4.2(b), 4.2(c) shows the $f'(\eta)$, $\theta(\eta)$ and $\Phi(\eta)$ profiles for varying magnetic parameter $M$ This points to a shrinkage of the thermal boundary layer. There is a correlation between $M$ and the temperature increase, which is shown in the graph. This is because a rise in $M$ causes particles in the fluid to cease their motion, causing rapid diffusion into the surrounding fluid layers. The velocity and concentration profiles shows decreasing nature because of opposing force named as Lorentz force.

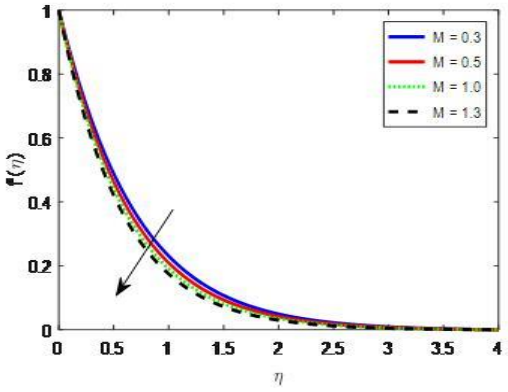


Figure 4.2 (a). $f'(\eta)$ profiles against $M$.

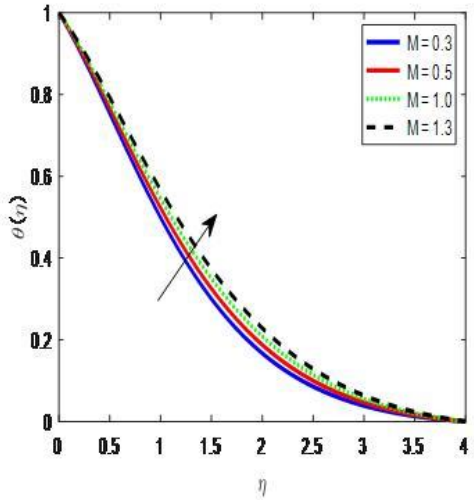


Figure 4.2 (b). $\theta(\eta)$ profiles against $M$.

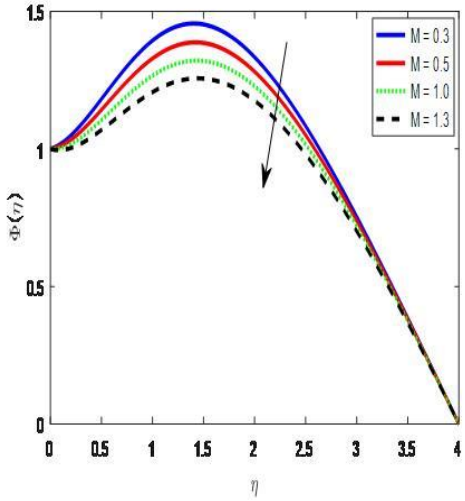


Figure 4.2 (c). $\Phi(\eta)$ profiles against $M$.

**4.3 Effect of Double Stratification on MHD Williamson Boundary Layer Flow and Heat Transfer across a Shrinking / Stretching Sheet Immersed in a Porous Medium**

The present section aims to provide a mathematical model of the Williamson fluid flow via a permeable stretching/shrinking sheet in the MHD boundary layer in the presence of a heat source, chemical reaction, and suction. This study is novel because it investigates the physical effects of thermal and solutal stratification on convective heat and mass transport using thermal radiation. The flow's PDEs are numerically solved using the bvp4c approach and the pertinent similarity variables until a stable solution is found. Through visual analysis, the effects of dimensionless factors on temperature, velocity, and concentration profiles are examined. This encompasses the mass transfer rate, the heat transfer rate, and the coefficient of friction. The results of the present analysis are found to be consistent with those of previous published studies. The findings demonstrate that enhanced temperature and concentration profiles cause the Williamson, magnetic, and permeability parameters to rise in conjunction with a drop in the dimensionless velocity. In relation to temperature, the thermal stratification parameter exhibits the opposite tendency. Regarding the solutal stratification parameter, concentration profiles are seen to show the opposite trend. Lastly, the current work will have important implications for the removal of dust and viruses from viscoelastic fluid in bioengineering, the medical sciences, and medical equipment.

The results of the Williamson parameter on the concentration, temperature, and velocity profiles during expansion and contraction are shown in Figures 4.3(a), 4.3(b), 4.3(c). The graph shows that increasing Γ causes fluid velocity to drop while increasing Γ causes temperature and concentration distributions to climb. Physically, a higher Γ decreases the fluid's velocity and enhances the fluid's heat distribution through the boundary layer by increasing the viscous forces that hold the Williamson fluid layers together. Fluid motion is impeded by the extended relaxing time. In physical studies of viscoelastic flows, the Γ is used to measure the contribution of elastic to viscous forces.

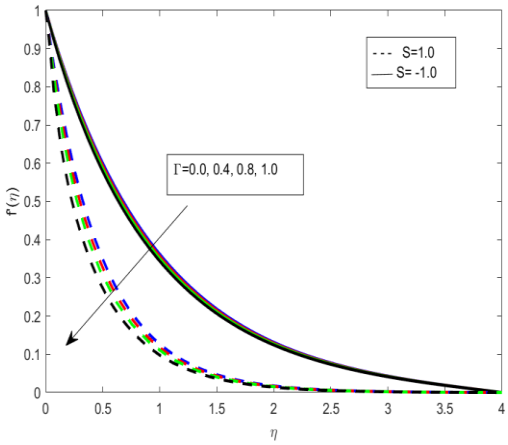


Figure 4.3(a). Outcome of Γ on velocity profiles.

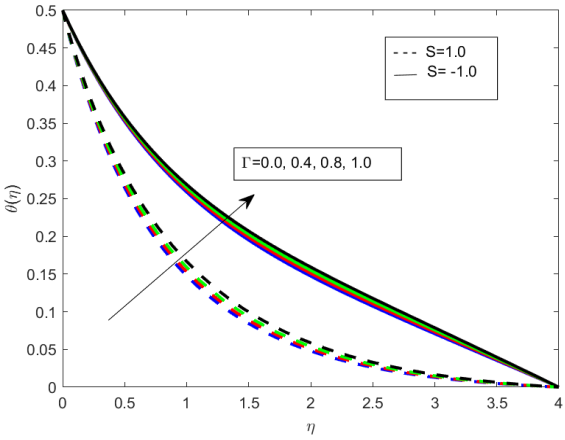


Figure 4.3(b). Outcome of Γ on temperature profiles.

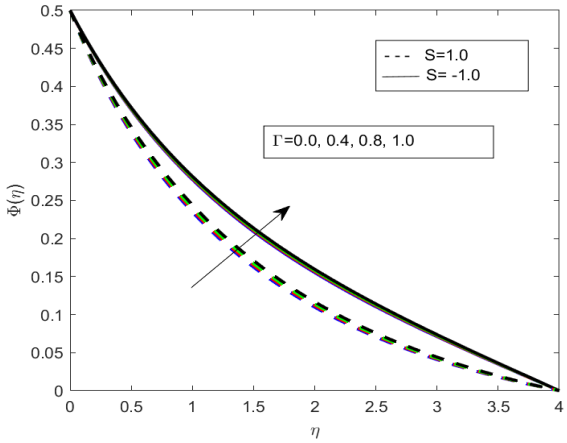


Figure 4.3(c). Outcome of Γ on Concentration profiles.

### 4.4 Heat and Mass Transfer Analysis of Unsteady MHD Carreu Nanofluid Flow over a Stretched Surface in a Porous Medium with Stefan Blowing Condition

The magneto-hydrodynamic (MHD) flow of a non-Newtonian nanofluid across an unstable stretched surface is thoroughly examined in this section, with special attention to the effects of suction or Stefan blowing. Innovative methods are used, such as assuming a fluid duplication in two phases for the nanofluid and using the Carreau fluid representation for the non-Newtonian fluid. The equations for the Carreau fluid flow model are transformed into dimensionless non-linear ordinary differential equations by using similarity transformations. The complex MATLAB programme bvp4c is used to acquire the numerical solutions to these equations. In This section investigates the effects of many controlling dimensionless parameters on mass transfer, temperature, concentration, friction factor, and dimensionless velocity. Graphs and tables are used to efficiently visualise the results, enabling in-depth study and debate. Notably, temperature and fluid velocity grow simultaneously with an increase in the Stefan blowing/suction parameter. Conversely, the temperature experiences a decline with increasing fluid velocity and Weissenberg number. These significant observations hold immense implications for the functionality and durability of crucial equipment, such as bearings, sliding components, and engines. By reducing thermal resistance and enhancing the heat transfer coefficient, Stefan blowing demonstrates its potential in significantly improving heat transfer effectiveness. The combination of Carreau nanofluid and Stefan blowing showcases an exceptional synergy, offering promising avenues for applications in cooling systems, thermal management tools, and lubricating capabilities within the oil and gas sector. The findings contribute to advancements in thermal management and offer a unique perspective on engineering applications across various industries.

The differences in dimensionless velocity, temperature, and concentration profiles with respect to power law index parameter n are shown in Figures 4.4(a), 4.4(b), 4.4(c). These data clearly confirm that the velocity and concentration decrease with increasing n, but the temperature distribution behaves in the opposite trend for both the Stefan blowing and suction instances. This is due to the fact that the power-law index increases the resistive force and, as a result, the fluid's non-Newtonian behavior by decreasing the sheet's non-linearity.

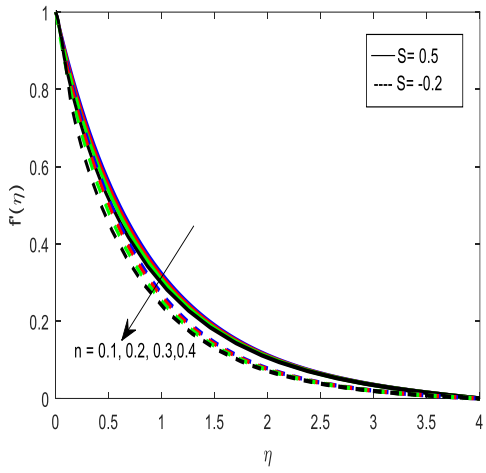


Figure 4.4(a). Differences in $f'(\eta)$ for various $n$ values.

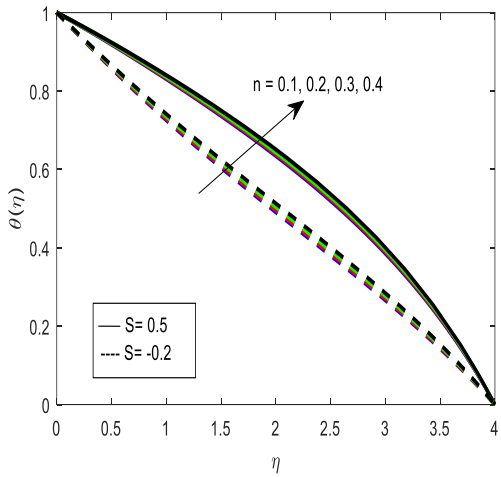


Figure 4.4(b). Differences in $\theta(\eta)$ for various $n$ values.

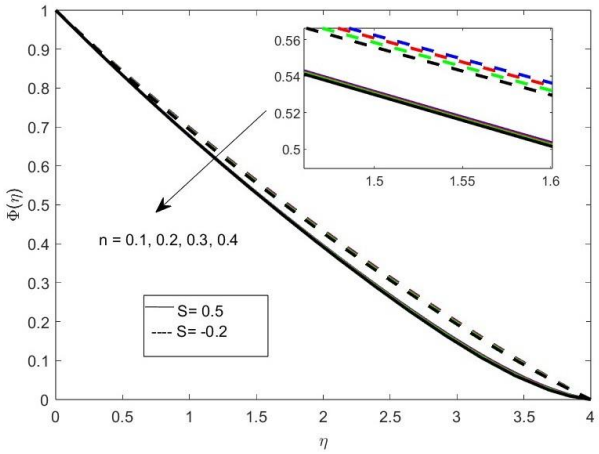


Figure.4.4(c). Differences in $\Phi(\eta)$ for various $n$ values.

**4.5 Impact of Slip Boundary Conditions on Magnetohydrodynamic Jeffrey Fluid Flow in a Tapered Asymmetric Channel with Soret-Dufour Effects**

This section provides the computational analysis and mathematical modeling of peristaltic blood flow in a tapered channel, taking into account slip boundary conditions, Hall current, and Soret-Dufour forces. By neglecting the wave number and employing a long-wavelength approximation to analyze the fluid model, the investigation was conducted within the framework of a low Reynolds number regime. The study reveals that the values of the Soret number, Dufour number, Prandtl number, and Schmidt number exhibit an upward trend as the fluid temperature rises.

In contrast, a rise in the thermal slip parameter causes the fluid temperature to fall. It has been shown that when the mass slip parameter increases, the fluid's concentration rises. The skin friction coefficient rises within the range of x = 0.55 to x = 1 as the velocity, concentration slip parameter, soret number, and dufour number increase. Conversely, the skin friction coefficient decreases as the thermal slip parameter increases. The selected characteristics exhibit realism since they find use in many fields such as medical biology, biomechanics, heat exchangers, gas turbines, and several other domains.

The impact of Hall current ($m$) on the velocity of the fluid is shown in Figure 4.5 (a). This graph indicates that the fluid velocity enhances in the portion of the channel $x \in (-0.75, 0.75)$ with an increase in Hall current. Larger values of Hall current leading to a decrease in effective conductivity can indeed cause a decrease in magnetic damping force and consequently enhance the axial velocity. The impact of $M$ on fluid velocity is evident in Figure 4.5 (b). The data in the image indicates a negative relationship between the fluid's velocity and the Hartmann number ($M$). The fluid's velocity decreases with increasing Hartmann number. Because of the Lorentz force operating on the fluid, its velocity drops as the Hartmann number increases.

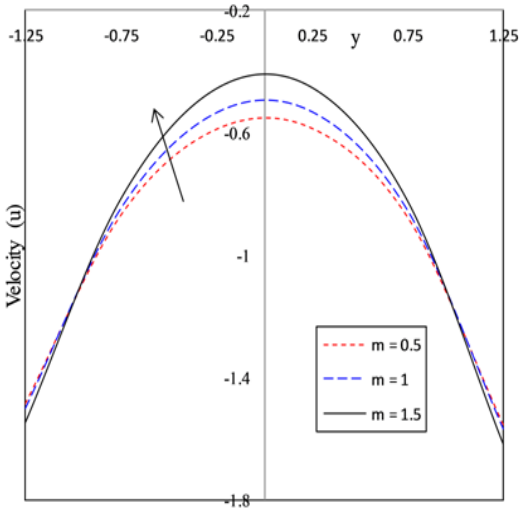


Figure 4.5 (a). Influence of m on u(y).

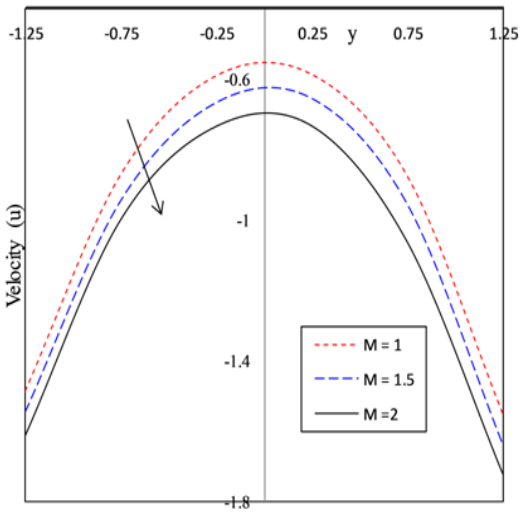


Figure 4.5 (b). Influence of $M$ on u(y).

## 5. ORGANIZATION OF THESIS

The thesis comprises of seven chapters.

**Chapter 1:**"Introduction" provides the core concepts of the current work, encompassing both the motivation behind the study and its research objectives.

**Chapter 2:** Explores the "Effect of Second Order Chemical Reaction on MHD Forced Convection Cu, Ag, and $Fe_3O_4$ Nanoparticles of Jeffrey Nanofluid over a Moving Plate in a Porous Medium in the Presence of Heat Source/Sink". In these chapters, the governing partial differential equations characterizing the fluid flow, coupled with the physical properties of the fluids, form the basis for solving the problem. Transformation of these partial differential equations (PDEs) into ordinary differential equations (ODEs) is facilitated through the application of suitable similarity transformations. MATLAB bvp4c and along with the shooting technique is employed to obtain numerical solutions for different physical quantities, enabling a thorough analysis of various flow properties. The local skin friction, the local Nusselt number, and the local Sherwood number were computed and displayed exhaustively. The results of the current study were compared numerically for various cases with earlier published results and found to be in good agreement.

**Chapter 3:** Portrays the "Nonlinear Coupled PDEs for Nanofluid Flow and Heat Transfer with Varying Thickness: A Comparative Study". The governing equations of the physical model are solved using Runge-Kutta shooting technique. The numerical computation of the solution of velocity, temperature and concentration are carried out for different non dimensional parameters of the problem. The skin friction coefficient, Nusselt number and local Sherwood number are calculated. The present results are compared with available existing results. It is found in good agreement with earlier works for the particular cases of the problem.

**Chapter 4:** Investigates the "Effect of Double Stratification on MHD Williamson Boundary Layer Flow and Heat Transfer Across a Shrinking / Stretching Sheet Immersed in a Porous Medium". In this chapter, the framed PDE's are transformed into ODE's by using similarity transformations. The final results are cracked numerically via bvp4c MATLAB package. The numerical computation of the solution of velocity, temperature and concentration are carried out for different non dimensional parameters of the problem. The skin friction coefficient, Nusselt number and local Sherwood number are calculated. The present results are compared with

available existing results. It is found in good agreement with earlier works for the particular cases of the problem.

**Chapter 5:** Focuses on investigating the "Heat and Mass Transfer Analysis of Unsteady MHD Carreu Nanofluid Flow over a Stretched Surface in a Porous Medium with Stefan Blowing Condition". In this chapter, the governing partial differential equations characterizing the fluid flow, coupled with the physical properties of the fluids, form the basis for solving the problem. Transformation of these partial differential equations (PDEs) into ordinary differential equations (ODEs) is facilitated through the application of suitable similarity transformations. MATLAB bvp4c and along with the shooting technique is employed to obtain numerical solutions for different physical quantities, enabling a thorough analysis of various flow properties. The local skin friction, the local Nusselt number, and the local Sherwood number were computed and displayed exhaustively. The results of the current study were compared numerically for various cases with earlier published results and found to be in good agreement. The impact of various parameters on velocity, temperature, concentration, nanoparticle volume fraction, skin friction, heat transfer, mass transfer, and nanoparticle mass transfer were presented through graphs.

**Chapter 6:** Investigates the "Consequence of Slip Boundary Conditions on Conducting Jeffrey Fluid Flow in a Tapered Vertical Asymmetric Channel with Hall Current, Soret and Dufour Impacts". Analytical solutions for the flow variables were derived by linearizing the momentum, temperature, and concentration equations under the assumptions of a long wavelength and a low Reynolds number. Skin friction and streamlines expressions have been acquired. For the purpose of analyzing the results, graphical representations are generated using Mathematica software. The findings can be used to improve hydraulic systems in engineering and gastrointestinal functions.

**Chapter 7:** Provides the general summary, conclusions of the thesis and outlines potential future directions for the research.

**REFERENCES**


1. S. Nadeem, N. S. Akbar, Effects of heat and mass transfer peristaltic flow of Williamson fluid in a vertical annulus, *Meccanica,* **47**(1) (2012), 141-151.
2. K. Vajravelu, S. Sreenadh, K. Rajanikanth, C. Lee, Peristaltic transport of a Williamson fluid in asymmetric channels with permeable walls, *Nonlinear Analysis: Real World Applications,***13**(6) (2012), 2804-2822.
3. I. Dapra, G. Scarpi, Perturbation solution for pulsatile flow of a nonNewtonian Williamson fluid in a rock fracture, *International Journal of Rock Mechanics and Mining Sciences,* **44**(2) (2007), 271-278.
4. N. Alam Khan, H. Khan, A boundary layer flows of non-Newtonian Williamson fluid, *Nonlinear Eng.* **3**(2) (2014), 107–115.
5. S. Nadeem, S. T. Hussain, C. Lee, Flow of a Williamson fluid over a stretching sheet, Braz. *J. Chem. Eng.*, **30** (3) (2013), 619–625.
6. T. Hayat, U. Khalid, M. Qasim, Steady flow of a Williamson fluid past a porous plate, *Asia Pac. J. Chem. Eng.*,**7** (2012), 302–306.
7. B. Reddappa, M. Sudheer Babu, S. Sreenadh, P. Durgaprasad, The flow of non-Newtonian fluid in an inclined channel through variable permeability, *Heat Transfer*., **52**(4) (2023), 3058-3073.
8. M. Sudheer Babu, B. Reddappa, K. A. Ajmath, Chemical reaction effects on MHD heat and mass transfer of a Jeffrey fluid flow with viscous dissipation and joule heating through a porous stretching sheet, *ARPN Journal of Engineering and Applied Sciences,* **18**(2) (2023), 102-112.
9. T. S. Sreeremya, A. Krishnan, P. Mohamed, U. Hareesh, and S. Ghosh, Synthesis and characterization of cerium oxide based nanofluids: An efficient coolant in heat transport applications, *Chem. Eng. J.,* **255** (2014), 282–289.
10. E. Zarifi, G. Jahanfarnia, and F. Veysi, Sub channel analysis of nanofluids application to VVER-1000 reactor, *Chem. Eng. Res.*, **91** (2013), 625–632.
11. F. Su, Y. Deng, and H. Ma, Numerical analysis of ammonia bubble absorption in a binary nanofluid, *Chem. Eng. Commun.,* **202** (2015), 500–507.
12. D. Tripathi, and O.A. Bég, Mathematical modelling of peristaltic pumping of nano-fluids. In Modelling and Simulation of Diffusive Processes, *Springer:*

*Berlin/Heidelberg, Germany*, (2014), 69–95.

13. S. A. Khan, Y. Nie, and B. Ali, Multiple slip effects on MHD unsteady viscoelastic nano-fluid flow over a permeable stretching sheet with radiation using the finite element method, *SN Appl. Sci.*,**2**(2020),1–14.
14. L. Liu, Aggregation of silica nanoparticles in an aqueous suspension, *AIChE J.*,**61** (2015), 2136–2146.
15. B. Ali, S. Hussain, S. I. R. Naqvi, D. Habib, and S. Abdal, Aligned magnetic and bio-convection effects on tangent hyperbolic nanofluid flow across faster/slower stretching wedge with activation energy: Finite element simulation, *Int. J. Appl. Comput. Math.*,**7** (2020),1–20.
16. B. Ali, G. Rasool, S. Hussain, D. Baleanu, and S. Bano, Finite element study of magnetohydrodynamics (MHD) and activation energy in Darcy–Forchhimer rotating flow of Casson Carreau nanofluid, *Processes(MDPI),* **8** (2020), 1185.
17. H. Masuda, A. Ebata, K. Teramae, N. Hishinuma, and Y. Ebata, Alteration of thermal conductivity and viscosity of liquid by dispersing ultra-fine particles (dispersion of-$Al_2O_3$, $SiO_2$ and $TiO_2$ ultra-fine particles). *NETSU BUSSEI,* **7** (1993), 227–233.
18. S. Choi, Enhancing thermal conductivity of fluids with nanoparticle, in: Development and Applications of Non-Newtonian Flow, *ASME Fluids Eng. Div.*, **231** (1995), 99–105.
19. J. Buongiorno, Convective transport in nanofluids, *J. Heat Transf.*, **128** (2006), 240–250.
20. J. A. Eastman, S. Phillpot, S. Choi, and P. Keblinski, Thermal transport in nanofluids, *Annu. Rev. Mater. Res.*, **34** (2004), 219–246.
21. M. Khairul, K. Shah, E. Doroodchi, R. Azizian, and B. Moghtaderi, Effects of surfactant on stability and thermo-physical properties of metal oxide nanofluids, *Int. J. Heat Mass Transf.,* **98** (2016), 778–787.
22. B. Ali, Y. Nie, S. Hussain, D. Habib, and S. Abdal, Insight into the dynamics of fluid conveying tiny particles over a rotating surface subject to Cattaneo–Christov heat transfer, Coriolis force, and Arrhenius activation energy, *Comput. Math. Appl.*, **93** (2021), 130–143.

23. S. A. Bagherzadeh, E. Jalali, M. M. Sarafraz, O. A. Akbari, A. Karimipour, M. Goodarzi, and Q.V. Bach, Effects of magnetic field on micro cross jet injection of dispersed nanoparticles in a microchannel, *Int. J. Numer. Methods Heat Fluid Flow,* **30** (2019), 2683–2704.
24. G. B. Jeffery, The Two-Dimensional Steady Motion of a Viscous Fluid, *Nelineinaya Din.,***29**(172) (2009), 101–109.
25. G. Hamel, Spiralförmige Bewegungen zäher Flüssigkeiten, *Jahresbericht der Dtsch. Math.* **25** (1917), 34–60.
26. V. E. Pohlhausen, Berechnung der Eigenschwingungen statiisch besiimmter Fachwerke, *Zeitsclirilt fiir rtngewatidte Mdthemstik und Mech.* **1** (1) (1921), 28–42.
27. K. Millsaps, K. Pohlhausen, Thermal Distributions in Jeffery-Hamel Flows Between Nonparallel Plane Walls, *J. Aeronaut. Sci.* **20** (3) (1953), 187–196.
28. W. T. Harrison, The Pressure in a Viscous Liquid Moving Through a Channel with Diverging Boundaries, *In Proc. Camb. Phil. Soc.,* **19** (1920), 307–312.
29. W. Tollmien, Gnenzschichtheoric Handbuch der Exper imental Physik, *Akadeneisahe Verlagsgesellschaft,* **4**(1) (1921), 241.
30. F. Noether. Handbuch der physikalischen und technischen Mechanik, *Nature,* **133** (3361) (1934), 480–480.
31. W. R. Dean, LXXII. Note on the divergent flow of fluid, *London, Edinburgh, Dublin Philos, Mag. J. Sci.,* **18** (121) (1934), 759–777.
32. K. Bhattacharyya, G.C. Layek, MHD boundary layer flow of dilatant fluid in a divergent channel with suction or blowing, *Chinese Phys. Lett.* **28** (8) (2011), 84705.
33. A. S. Dogonchi, D.D. Ganji, Investigation of MHD nanofluid flow and heat transfer in a stretching/shrinking convergent/divergent channel considering thermal radiation, *J. Mol. Liq.,* **220** (2016), 592–603.
34. N. G. Sravan, S. K. Shravan, The effect of Soret parameter on the set of double diffusive convection on Darcy porous medium saturated with couple stress fluid, *Advances in Applied Science Research,* **3** (2012), 1426-1434.
35. M. S. Alam, M. M. Rahman, Dufour and Soret effects on mixed convection flow past a vertical porous flat plate with variable Suction, *Non Linear*

*Analysis: Modeling and Control,***11**(1)(2006), 3-12.

36. A. Postelnicu, Influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in a porous media considering Soret and Dufour effects, *International Journal of Heat and Mass Transfer*, **44** (2004), 1467-1472.
37. M. S. Alam, M. M. Rahman, M. A. Samad, Dufour and Soret effects on unsteady MHD free convection and mass transfer flow past a vertical porous plate in a porous medium, *Nonlinear Analysis: Modeling and Control*, **11** (2006), 271-226.
38. M. S. Alam, M. M. Rahman, Dufour and Soret effects on MHD free convective heat and mass transfer flow past a vertical flat plate embedded in a porous medium, *Journal of Naval Architecture and Marine Engineering*, **2**(1) (2005), 55-65.
39. A. Afsar Khan, R. Ellahi, K. Vafai, Peristaltic Transport on a Jeffery Fluid with Variable Viscosity Through a Porous Medium in Asymmetric Channel, *Advances in Mathematical Physics*, **10**(1155) (2012), 1-15.
40. B. Reddappa, S. Sreenadh, K. Kumaraswamy Naidu, and A. Parandhama, Poiseuille Flow of Conducting Jeffrey Fluid Between Parallel Plates When one of the Walls is Provided with Porous Lining, *Ultra Scientist*, **28**(1)A (2016), 71-78.
41. B. Reddappa, S. Sreenadh, K. Kumaraswamy Naidu, Convective Couette flow of a Jeffrey fluid in an inclined channel when the walls are provided with porous lining, *Advances in Applied Science Research*,**6**(12) (2015), 69-75.

**LIST OF PUBLICATIONS**

1. B. Reddappa, **R. Geetha**, Effects of second order chemical reaction on MHD forced convection Cu, Ag, and $Fe_3O_4$ nanoparticles of Jeffrey Nanofluid over a moving plate in a porous medium in the presence of heat source/sink, *Journal of Integrated Science & Technology*, **12**(3) 762, 2024.
2. B. Reddappa, **R. Geetha**, B. Rushi Kumar, Mehdi Akermi, M. Ijaz Khan and Ilyas Khan, Nonlinear coupled ODEs for nanofluid flow and heat transfer with varying thickness: A comparative study, *Numerical Heat Transfer, Part B:Fundamentals,* 2024.
3. **R. Geetha**, B. Reddappa, Nainaru Tarakaramu, B. Rushi Kumar, M. Ijaz Khan, Effect of double stratification on MHD Williamson boundary layer flow and heat transfer across a shrinking / stretching sheet immersed in a porous medium, *International Journal of Chemical Engineering* , **2024**(9983489), 2024.
4. **Geetha Ramakrishnan**, Reddappa Bandi, Ravikumar Seelam, Lakshminarayana Pallavarapu, Influence of Slip Boundary Conditions on Jeffrey Fluid Flow in Tapered Conduit with Hall Current and Soret-Dufour Effects, *Chemical Engineering & Technology,* 2024.